Solution Sheet

1.

$$f(x) = 4x^{\frac{1}{2}} + 4x^{-\frac{1}{3}}$$

$$\begin{aligned} f'(x) &= 4 \times \frac{1}{2}x^{\frac{1}{2}-1} - 4 \times \frac{1}{3}x^{-\frac{1}{3}-1} \\ &= 2x^{-\frac{1}{2}} - \frac{4}{3}x^{-\frac{4}{3}} \end{aligned}$$

2.

$$y = 5x^3 + x^{\frac{1}{2}} - 3$$

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= 5 \times 3x^{3-1} + \frac{1}{2}x^{\frac{1}{2}-1} \\ &= 15x^2 + \frac{1}{2}x^{-\frac{1}{2}} \end{aligned}$$

3.

$$y = 2x^{-\frac{1}{3}} - x^{-3}$$

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= 2 \times \left(-\frac{1}{3}\right) x^{-\frac{1}{3}-1} - (-3)\, x^{-3-1} \\ &= -\frac{2}{3}x^{-\frac{4}{3}} + 3x^{-4} \end{aligned}$$

4.

$$y = x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 4$$

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{2}{3}x^{\frac{2}{3}-1} - 5 \times \frac{1}{3}x^{\frac{1}{3}-1} \\ &= \frac{2}{3}x^{-\frac{1}{3}} - \frac{5}{3}x^{-\frac{2}{3}} \end{aligned}$$

5.

$$y = 3x^{\frac{1}{3}} - 2x^{\frac{2}{3}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 \times \frac{1}{3}x^{\frac{1}{3}-1} - 2 \times \frac{2}{3}x^{\frac{2}{3}-1}$$
$$= x^{-\frac{2}{3}} - \frac{4}{3}x^{-\frac{1}{3}}$$

6.

$$y = 2x^3 - 2x^{-\frac{2}{3}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 \times 3x^{3-1} - 2 \times \left(-\frac{2}{3}\right) x^{-\frac{2}{3}-1}$$
$$= 6x^2 + \frac{4}{3}x^{-\frac{5}{3}}$$

7.

$$f(x) = 3x^{-2} + 5x^{-1} - 4$$

$$f'(x) = 3 \times (-2)\, x^{-2-1} - 5x^{-1-1}$$
$$= -6x^{-3} - 5x^{-2}$$

8.

$$y = 6x^{-2} - 2x^{\frac{1}{3}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6 \times (-2)\, x^{-2-1} - 2 \times \frac{1}{3}x^{\frac{1}{3}-1}$$
$$= -12x^{-3} - \frac{2}{3}x^{-\frac{2}{3}}$$

9.

$$y = x^{\frac{1}{2}} + 3x^{\frac{3}{2}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x^{\frac{1}{2}-1} + 3 \times \frac{3}{2}x^{\frac{3}{2}-1}$$
$$= \frac{1}{2}x^{-\frac{1}{2}} + \frac{9}{2}x^{\frac{1}{2}}$$

10.

$$f(x) = x + 5x^{\frac{1}{2}}$$

$$\begin{aligned} f'(x) &= 1 + 5 \times \frac{1}{2}x^{\frac{1}{2}-1} \\ &= 1 + \frac{5}{2}x^{-\frac{1}{2}} \end{aligned}$$

11.

$$f(x) = 4x^{\frac{3}{2}} - 2x$$

$$\begin{aligned} f'(x) &= 4 \times \frac{3}{2}x^{\frac{3}{2}-1} - 2 \\ &= 6x^{\frac{1}{2}} - 2 \end{aligned}$$

12.

$$f(x) = 2x^{-3} - 5x^3 + 4$$

$$\begin{aligned} f'(x) &= 2 \times (-3)\, x^{-3-1} - 5 \times 3x^{3-1} \\ &= -6x^{-4} - 15x^2 \end{aligned}$$

13.

$$y = 2x^{\frac{2}{3}} - 2x^2$$

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= 2 \times \frac{2}{3}x^{\frac{2}{3}-1} - 2 \times 2x^{2-1} \\ &= \frac{4}{3}x^{-\frac{1}{3}} - 4x \end{aligned}$$

14.

$$y = 6x + 2x^{\frac{1}{2}} - 5$$

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= 6 + 2 \times \frac{1}{2}x^{\frac{1}{2}-1} \\ &= 6 + x^{-\frac{1}{2}} \end{aligned}$$

15.

$$y = 4x^{\frac{2}{3}} - 4x^{\frac{1}{2}}$$

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} &= 4 \times \frac{2}{3}x^{\frac{2}{3}-1} - 4 \times \frac{1}{2}x^{\frac{1}{2}-1} \\ &= \frac{8}{3}x^{-\frac{1}{3}} - 2x^{-\frac{1}{2}}\end{aligned}$$